1. Experimental Settings

In this section, the two different dynamic reference point specification mechanism (i.e., the linearly decreasing mechanism and the weak convergence detection mechanism) are tested with the state-of-the-art algorithm SMS-EMOA [7]. The DTLZ test suite [12], WFG test suite [31], their minus-versions [32], and Multi-Point Distance Minimization Problem (MPDMP) are used in this experiment. We consider the problems with 10 objectives. All the code in this section is implemented in PlatEMO framework [21] with the following settings:

Population size: 30 (*H*=1),

Total evaluation number: 100,000 solution evaluations,

Initial value of r ()： 10，

Crossover: Simulated binary crossover with probability of 1.0,

Mutation: Polynomial mutation with probability of 1/*D*,

Distribution index of crossover and mutation: 20,

Number of decision variables, *D*: 14 (DTLZ1 and minus-DTLZ1),

2 (MPDMP),

19 (other problems).

1. Computational Results

In our experiments, four versions of SMS-EMOA algorithm with different reference point specification mechanisms are considered. These algorithms are named as SMS-EMOA-10, SMS-EMOA-Opt, SMS-EMOA-LD, and SMS-EMOA-CD. For SMS-EMOA-10, the value r is set to 10. The value r for SMS-EMOA-opt is set to . SMS-EMOA with the linearly decreasing mechanism and with the weak convergence detection mechanism are referred to as SMS-EMOA-LD and SMS-EMOA-CD, respectively. The HV value after 100,000 evaluations are obtained for each algorithm. Table I and Table II show the computational results for HV metrics. The best result in each row is highlighted in bold, and the worst result is shaded. The Wilcoxon rank sum test is used to show the statistical significance for SMS-EMOA-10, SMS-EMOS-Opt, SMS-EMOA-LD in comparison to our proposed SMS-EMOA-CD. The three symbols “+”, “-”, “” mean significantly better, significantly worse and no significant difference.

In Table I, the results show that SMS-EMOA-Opt performs the worst (9 out of 13 significantly worse than SMS-EMOA-CD) among the algorithms. This result shows that when applying all the time, bad searching behaviour is obtained. We can not tell the differences between SMS-EMOA-10 and SMS-EMOA-Opt, as the Wilcoxon rank sum tests show almost all the results are “”. This is probably because of the small influence of reference point on the triangular PF problems. As for SMS-EMOA-LD, two better results are obtained when compared to SMS-EMOA-CD. This indicate that SMS-EMOA-LD is slightly better than SMS-EMOA-CD on triangular PF problems.

Table II shows the results obtained for inverted-triangular PF problems (i.e., minus-DTLZ1-4, minus WFG1-9, and MPDMP). The results show that SMS-EMOA-10 performs the worst (12 out of 14) on most of the inverted-triangular PF problems, and the Wilcoxon rank sum tests show that almost all the results from SMS-EMOA-10 are significantly worse than SMS-EMOA-CD (11 out of 14 worse and no better results). This result can be explained by the setting of r. Since the value of r is set to 10 for the whole process of SMS-EMOA-10, many solutions are on the boundary of PF. （我感觉这里还要多一点解释，可能一句话囊括为何solutions on the boundary of PF会导致差的表现）As compared to SMS-EMOA-CD, the performances of SMS-EMOA-Opt (2 better but 4 worse results) and SMS-EMOA-LD (2 better but 3 worse results) are slightly worse on inverted-triangular PF problems. The results of MPDMP shows that SMS-EMOA-CD is the best among the four algorithms.

Fig. 7 shows the plot of HV for the four mechanisms on the 10-dimensional MPDMP problem. In Fig.7, the HV of SMS-EMOA-LD (the red curve) gradually increases and finally reaches the same level as SMS-EMOA-Opt, as the value of r is gradually decreased to 1+1/H. The HV of SMS-EMOA-CD (the yellow curve) firstly reaches a stable level similar to SMS-EMOA-10, for that their values of r are both 10 before 4,500 evaluations. The convergence detection is reported at about 4,500 evaluations for SMS-EMOA-CD. Then, the values of r in SMS-EMOA-CD reaches the optimal value of 1+1/H and the HV increases. The reason is due to the decrease of the boundary solutions, and on the other hand, the inner solutions increase. Finally, the HV of SMS-EMOA-CD is better than SMS-EMOA-Opt and SMS-EMOA-LD.

Conclusion

In this paper, we emphasize the importance of reference point specification in SMS-EMOA by a simple example. We have demonstrated that without a good reference point specification mechanism, a poor diversity of the final solutions on inverted-shape PF problems will be obtained. This phenomenon is hardly observed on the triangular PF problems when the reference point is worse than the nadir point. We introduced the dynamic reference point specification mechanism with the illustration by two aspects:

1. Better searching behavior: In the early stage, the solutions may be far away from the true PF. A larger value of r can achieve better searching behavior. We give an example of DTLZ2.
2. Uniform distribution: Considering the linear triangular problems, the optimal setting of the reference point is r=1+1/H.

We summarize the basic idea of dynamic reference point specification mechanism as follows: the value of r should be specified larger than 1+1/H at the initial stage and equal to 1/1+H in the final stage, as in Eqs. (4)-(6).

In this paper, we have proposed a new dynamic reference point adaptation mechanism. A weak convergence detection mechanism is used in the proposed method. The new dynamic reference point adaptation mechanism is tested on SMS-EMOA algorithm and compared with other SMS-EMOA under different setting of r. The results show that SMS-EMOA with r=1+1/H performs the worst on the triangular PF problems, and SMS-EMOA with r =10 performs the worst on the inverted-triangular PF problems. This give us a hint on the necessary of the dynamic mechanism. We have also compared our proposed mechanism with the linearly decreasing mechanism. The results show that our new mechanism outperforms the linearly decreasing mechanism on some test problems, specifically on the MPDMP problem.

In the future, we plan to investigate the behaviour of our new mechanism and further improve it. Besides that, the problems with different PF shapes will be tested and analysed.